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Abstract

We address the problem of adjusting freight demand matrices to observed flows for multimodal, multicommodity transportation planning. We introduce a model and describe an algorithm to solve the corresponding bi-level optimization problem. The algorithm is efficiently implemented using STAN, an interactive-graphic transportation planning package designed for national or regional strategic analysis and planning of multimode multiproduct freight transportation. Data from an actual application is used to characterize the performance of the proposed methodology. We conclude with a number of generalizations and promising developments.

Key words: Freight Transportation, Demand Matrix Adjustment, Multimodal Strategic Planning, Path Analysis, Decision Support Systems

Résumé

Nous nous intéressons au problème de l'ajustement de matrices de demande à partir de flux observés de produits, à être utilisées dans le cadre de processus de planification multimode multiproduits de systèmes de transport de marchandises. Nous présentons une formulation d'optimisation bi-niveaux et décrivons un algorithme pour sa résolution. Nous présentons également une implantation efficace à l'aide du progiciel STAN dédié à la planification stratégique des réseaux nationaux ou régionaux de transport des marchandises. Des données provenant d'applications réelles servent à caractériser les performances de la méthodologie proposée. Nous concluons en décrivant des généralisations possibles du modèle et de l'algorithme présentés.

Mots clefs : Transport des marchandises, Ajustement de matrices de demande, Planification stratégique multimodale, Analyse de chemins, Systèmes d'aide à la décision

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1 Introduction

Transportation aims to supply the means to satisfy the needs in person and goods movements of human societies. It is thus not surprising that demand estimation and validation models and procedures form an essential component of all transportation planning methodologies.

Typically, the territory under study is divided into a number of *zones*, each represented in the network model of the transportation system by a special node called a *centroid*. Demand is then represented as the amplitude of movement between each pair of centroids: number of trips persons desire to undertake, number of tons of a given commodity, or group of commodities, to be shipped from one zone to another. Thus, the entire activity of a zone is aggregated into its corresponding centroid and centroid-to-centroid *demand matrices* capture the demand for transportation in the territory. Demand matrices are then assigned on the network representation of the transportation system. An optimization method is usually employed and the resulting *predicted* flows are the input to various analyses (Florian 1986; Florian and Hearn 1995; Crainic 1999, Crainic *et al.* 1990, 1999, 2000).

Many methods have been proposed to generate demand matrices, most of these targeting person transportation planning. Generating demand matrices is a long and costly process and the entire process is rarely repeated at regular intervals. Rather, existing matrices are updated, their *adjustment* to *observed flows* being one of the methods of choice (Cascetta and Nguyen 1988; Chen and Florian 1996). Observed flows are generally obtained by measuring the number of vehicles that pass during a given time period by a number of counting posts. Existing demand matrices are then adjusted such that the flows on the links of the network representation obtained following assignment are as close as possible to the observed flows.

In freight transportation planning the need for demand matrix evaluation and adjustment is as paramount as when passenger transportation is considered. Yet, there is little to be found in the published literature concerning this subject. The present paper constitutes a first step in closing the gap and answering the needs of freight transportation planning.

The demand adjustment problem may appear similar for person or freight transportation planning. Indeed, both usually have to be addressed in a multimodal context. The latter is, however, significantly more complex due to the additional dimension induced by the multiple products that are simultaneously assigned on the multimodal network. Thus, the objective of this paper is double: 1) present a freight demand matrix adjustment methodology, and 2) describe an efficient implementation using a commercial software for multimodal, multicommodity transportation planning.

The paper is organized as follows. Section 2 introduces the freight demand matrix adjustment problem when observed flows are available for one product only. Section 3 describes the algorithm proposed to solve the corresponding bi-level optimization problem. Section 4 is dedicated to the presentation of an efficient implementations using the commercial freight transportation planning software STAN and the analysis of its performance using actual application data. Section 5 discusses two generalizations to cases where matrices of several products are simultaneously modified according to observed flows. We conclude in Section 6.

2 The Freight Demand Adjustment Problem

To formally describe the freight demand adjustment problem, one needs a representation of the multimodal, multiproduct transportation system and an associated assignment methodology. We adopt in the following the notation and approach introduced by Guélat, Florian, and Crainic (1990; see also Crainic 1999).

The multimodal network model consists of the nodes $\mathcal{N}$, modes $\mathcal{M}$, links $\mathcal{A}$, and intermodal transfers $\mathcal{T}$ required to represent the possible movements induced by the transportation demand on the available (and, eventually, planned) infrastructure. A mode $m \in M$ represents a possible means of transportation (e.g., trucking), a service (e.g., commodity-dedicated unit train on rail), or infrastructure (e.g., port). A link $a \in \mathcal{A}$ is defined as a triplet (i, m, j), where $i \in \mathcal{N}$ is the origin node, $j \in \mathcal{N}$ is the destination node, and $m \in \mathcal{M}$ is the mode allowed on the arc. An intermodal transfer $t \in \mathcal{T}$ at a node $j \in \mathcal{N}$ represents allowed movements from an incoming link (i, m_{in}, j) to an outgoing link (j, m_{out}, k). A product is any commodity or collection of similar goods, that generates a network flow. Products $p \in \mathcal{P}$ are transported over the multimode network. The volume of product p to be shipped from an origin $o \in \mathcal{N}$ to a destination $d \in \mathcal{N}$ forms the (o, d) cell of demand matrix g^p. The set of modes that may be used to move product p is denoted $m(p) \subseteq \mathcal{M}$.

Flows of product $p \in \mathcal{P}$ on links $a \in \mathcal{A}$ are denoted by v_a^p and flows on transfers $t \in \mathcal{T}$ are denoted by v_t^p. Unit cost functions $s_a^p(v)$ and $s_t^p(v)$, for a given flow vector v, are associated to the links and transfers of the network for each product p. To simplify the presentation, we assume that $s(v)$ is link and transfer separable, positive, and continuously differentiable. We also suppose that the total cost $F(v)$, defined by equation (1), is convex. The model may then be written as follows:

$$\text{Minimize} \quad F(v) = \sum_{p \in P} \left(\sum_{a \in \mathcal{A}} s_a^p(v_a) v_a^p + \sum_{t \in \mathcal{T}} s_t^p(v_t) v_t^p \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{l \in \mathcal{K}_{od}^p} h_l = g_{od}^p, \qquad o,d \in \mathcal{N},\ p \in \mathcal{P}, \tag{2}$$

$$h_l \geq 0, \qquad l \in \mathcal{K}, \tag{3}$$

$$v_a^p = \sum_{o \in \mathcal{N}} \sum_{d \in \mathcal{N}} \sum_{l \in \mathcal{K}_{od}^p} \delta_{al} h_l, \quad a \in \mathcal{A},\ p \in \mathcal{P}, \tag{4}$$

$$v_t^p = \sum_{o \in \mathcal{N}} \sum_{d \in \mathcal{N}} \sum_{l \in \mathcal{K}_{od}^p} \delta_{tl} h_l, \quad t \in \mathcal{T},\ p \in \mathcal{P}, \tag{5}$$

where, $\mathcal{K}_{od}^p$ denotes the set of paths that for product p lead from origin o to destination d using only modes in $m(p)$ ($\mathcal{K} = \bigcup_p \bigcup_{(o,d)} \mathcal{K}_{od}^p$); h_l is the flow on path $l \in \mathcal{K}_{od}^p$; $\delta_{al} = 1$ if $a \in l$ (and 0, otherwise) and $\delta_{tl} = 1$ if $t \in l$ (and 0, otherwise) are the indicator functions which identify the arcs and transfers, respectively, of a particular path. A multimodal path in this network model thus consists in a sequence of directed links and, eventually, transfers defined on the modes of a given mode choice set.

Flows may be observed on links and transfers. These measures may be obtained from counting posts that capture the number of vehicles of certain types that travel on specific links, or from more in-depth interviews conducted at appropriate locations such as toll booths, inspection stations, border crossings, etc. Observed flows may also be captured at transshipment or consolidation facilities: ports, rail yards, intermodal platforms, etc. In this case, tonnage rather than number of vehicles is the usual measure. Observed flows may thus represent a specific product or an aggregation of several commodities (e.g., when only vehicle observations are made). Moreover, given the multimodal nature of many freight itineraries, observations for the same product may be made of links of different modes.

Let us focus on the simple case where observed flows are available for one product only, $c \in \mathcal{P}$. The initially available demand matrix is g^c. Flows are observed and counted on some of the links and transfers of the network, $\hat{v}_a^c,\ a \in \widehat{\mathcal{A}} \subseteq \mathcal{A}$ and $\hat{v}_t^c,\ t \in \widehat{\mathcal{T}} \subseteq \mathcal{T}$, respectively; $\hat{v} = (\hat{v}_a^c, \hat{v}_t^c)$. Let $\mathcal{OD}^c$ represent the set of origin-destination pairs that are to be adjusted according to the observed flows, while $\hat{g}^c$ denotes the matrix resulting from the adjustment procedure.

The matrix adjustment process aims to modify the values of g^c, yielding $\hat{g}^c$, such that the link and transfer volumes in $v^\star$, corresponding to the assignment of $\hat{g}^c$, are as close as possible to the observed flow values in $\hat{v}$. In optimization terms, the problem can be stated as:

> Identify $g^\star = \arg\min(v^\star(g) - \hat{v})^2$, where $v^\star$ and $v^\star(g)$ stand for the optimal flow vector solution and associated objective function value to problem (1) to (5) for fixed demand matrix g.

Not any matrix will do, however. Indeed, as noted by several authors (e.g., Spiess

1990), this formulation admits a possibly infinite number of optimal solutions where each demand matrix reflects observed volumes equally well. In actual practice, however, demand matrices already exist and one generally desires that the new, adjusted matrix be "similar" to the initial matrix. Then, following Spiess (1990) and Florian and Chen (1995), we may write the general formulation of the matrix adjustment problem as follows:

$$\text{Minimize } G(g,v) = \frac{1}{2}\left[\sum_{od\in\mathcal{OD}^c}(g^c_{od}-\hat{g}^c_{od})^2 + \sum_{a\in\hat{\mathcal{A}}}(v^c_a-\hat{v}^c_a)^2 + \sum_{t\in\hat{\mathcal{T}}}(v^c_t-\hat{v}^c_t)^2\right] \tag{6}$$

$$\text{s.t. } g^c_{od}\geq 0, \quad od\in\mathcal{OD}^c \tag{7}$$

$$(\ldots,\ v^c,\ldots) = \arg\min\{F(g) \text{ constrained by (2) to (5)}\} \tag{8}$$

where relation (8) indicates that v^c is the component for product c of v^*, the optimal solution to problem (1) to (5) for fixed demand g.

3 A Steepest-Descent Algorithm

Let w^c_{odl}, $l\in\mathcal{K}^c_{od}$, represent the *proportion of the total demand of origin-destination pair od moved on path l*, with $h_l = \sum_{l\in\mathcal{K}^c_{od}} w^c_{odl}g^c_{od}$. When path flows for product c are expressed in terms of the w^c_l, $l\in\mathcal{K}^c$, the proportions of demand they represent, the demand matrix adjustment problem may be reformulated as follows:

$$\text{Minimize } G(g,w) = \frac{1}{2}\left[\sum_{od\in\mathcal{OD}^c}(g^c_{od}-\hat{g}^c_{od})^2 + \sum_{a\in\hat{\mathcal{A}}}\left(\sum_{od\in\mathcal{OD}^c}\sum_{l\in\mathcal{K}^c_{od}}\delta_{al}w^c_l g^c_{od}-\hat{v}^c_a\right)^2 + \sum_{t\in\hat{\mathcal{T}}}\left(\sum_{od\in\mathcal{OD}^c}\sum_{l\in\mathcal{K}^c_{od}}\delta_{tl}w^c_l g^c_{od}-\hat{v}^c_t\right)^2\right] \tag{9}$$

$$\text{s.t. } F(g^c,v)-\bar{F}(g^c)=0 \tag{10}$$

$$\sum_{l\in\mathcal{K}^c_{od}} w^c_l g^c_{od} = g^c_{od}, \quad od\in\mathcal{OD}^c,\ c\in\mathcal{P} \tag{11}$$

$$g^c_{od}\geq 0, \quad od\in\mathcal{OD}^c,\ c\in\mathcal{P} \tag{12}$$

$$w^c_l\geq 0, \quad l\in\mathcal{K}^c_{od},\ od\in\mathcal{OD}^c,\ c\in\mathcal{P} \tag{13}$$

where the marginal function $\bar{F}(g)$ is given by

$$\bar{F}(g) = \inf\{(1) \text{ constrained by (2) to (5)}\}. \tag{14}$$

Note that in terms of paths weights

$$v_a = \sum_{p \neq c} v_a^p + \sum_{od \in \mathcal{OD}^c} \sum_{l \in \mathcal{K}_{od}^c} \delta_{al} w_l^c g_{od}^c \tag{15}$$

$$v_t = \sum_{p \neq c} v_t^p + \sum_{od \in \mathcal{OD}^c} \sum_{l \in \mathcal{K}_{od}^c} \delta_{al} w_l^c g_{od}^c, \tag{16}$$

and

$$\begin{aligned} F(g^c, w^c) = & \sum_{a \in \mathcal{A}} \left[\sum_{p \neq c} s_a^p(v_a) v_a^p + s_a^c(v_a) \sum_{od \in \mathcal{OD}^c} \sum_{l \in \mathcal{K}_{od}^c} \delta_{al} w_l^c g_{od}^c \right] \\ & + \sum_{t \in \mathcal{T}} \left[\sum_{p \neq c} s_t^p(v_t) v_t^p + s_t^c) v_t) \sum_{od \in \mathcal{OD}^c} \sum_{l \in \mathcal{K}_{od}^c} \delta_{tl} w_l^c g_{od}^c \right]. \end{aligned} \tag{17}$$

Then, using convexity arguments similar to those of Florian and Chen (1995), one may show that $\bar{F}(g)$ is differentiable and

$$\begin{aligned} \nabla \bar{F}(g_{od}^c) = & \sum_{a \in \mathcal{A}} \left[\sum_{p \neq c} \frac{\partial s_a^p(v_a)}{\partial g_{od}^c} v_a^p + \frac{\partial s_a^c(v_a)}{\partial g_{od}^c} v_a^c + s_a^c(v_a) \sum_{l \in \mathcal{K}_{od}^c} \delta_{al} w_l^c \right] \\ & + \sum_{t \in \mathcal{T}} \left[\sum_{p \neq c} \frac{\partial s_t^p(v_t)}{\partial g_{od}^c} v_t^p + \frac{\partial s_t^c(v_t)}{\partial g_{od}^c} v_t^c + s_t^c(v_t) \sum_{l \in \mathcal{K}_{od}^c} \delta_{tl} w_l^c \right], \quad od \in \mathcal{OD}^c. \end{aligned} \tag{18}$$

The bi-level matrix adjustment optimization problem can then be solved by a *steepest descent*-type of algorithm as follows (see Florian and Chen, 1995, for justifications):

Initialization Solve the lower level assignment problem (1) to (5) with an initial demand matrix g^{c0} (usually, $g^{c0} \equiv g^c$). This yields initial link and transfer flows v_a^0 and v_t^0, as well as the corresponding path weights w^{c0}. $k = 0$.

Main Iteration Given g^{ck}, v_a^k, v_t^k, and w^{ck} computed during the previous iteration:

1. Compute the gradient of $G(g, w^{ck})$ at g^k:

$$\begin{aligned} b_{od}^{ck} = \left. \frac{\partial G(g, w^{ck})}{\partial g_{od}^c} \right|_{g_{od}^{ck}} & = g_{od}^{ck} - \hat{g}_{od}^c \\ & + \sum_{l \in \mathcal{K}_{od}^c} w_l^{ck} \left[\sum_{a \in \hat{\mathcal{A}}} \delta_{al} (v_a^{ck} - \hat{v}_a^c) + \sum_{t \in \hat{\mathcal{T}}} \delta_{tl} (v_t^{ck} - \hat{v}_t^c) \right] \quad od \in \mathcal{OD}^c. \end{aligned} \tag{19}$$

2. Compute the descent *gradient direction* d^k: $d_{od}^{ck} = -b_{od}^{ck} g_{od}^{ck}$.
   If $\nabla_g G(g^k, w^k) \cdot d^k \geq 0$, stop the algorithm. Otherwise
3. Compute the step length λ^k determined by

$$\lambda^k = \arg\min\{G(g^k + \lambda d^k) \text{ s.t. } \lambda b_{od}^{ck} \leq 1,\ od \in \mathcal{OD}^c\} \tag{20}$$

4. $g^{c(k+1)} = g^{ck} + \lambda^k d^k$
   $k = k + 1$.
5. Solve the lower level assignment problem with demand matrix g^{ck}, and go to Step 1.

The optimal value of the step length may be obtained by solving

$$\nabla_\lambda G(g^k + \lambda d^k) = \nabla_\lambda G(g^k + \lambda(x^k - g^k)) = 0. \tag{21}$$

From (9) and the usual chain-derivation rules $\lambda^k = \alpha^k / \beta^k$:

$$\alpha^k = \sum_{od \in \mathcal{OD}^c} (\hat{g}_{od}^c - g_{od}^{ck})\ d_{od}^{ck} + \sum_{a \in \hat{\mathcal{A}}} (\hat{v}_a^c - v_a^{ck})\ y_a^{ck}(d^k) + \sum_{t \in \hat{\mathcal{T}}} (\hat{v}_t^c - v_t^{ck})\ y_t^{ck}(d^k),$$

$$\beta^k = \sum_{od \in \mathcal{OD}^c} \left(d_{od}^{ck}\right)^2 + \sum_{a \in \hat{\mathcal{A}}} \left(y_a^{ck}(d^k)\right)^2 + \sum_{t \in \hat{\mathcal{T}}} \left(y_t^{ck}(d^k)\right)^2, \tag{22}$$

where $y_a^{ck}(d^k)$ and $y_t^{ck}(d^k)$ represent the link and transfer volumes that correspond to a distribution of d_{od}^{ck} volumes on the paths in $\mathcal{K}_{od}^{ck}$ according to the proportions w_l^{ck}. Since g^{k+1} must be non-negative, λ^k must verify $\lambda b_{od}^{ck} \leq 1,\ od \in \mathcal{OD}^c$.

Note that the usual steepest descent direction (in Step 2) is $-\nabla_g G(g, w^{ck})$. However, bending the direction by multiplying the gradient by the demand matrix contributes to the goal of avoiding over adjusting the matrix, while still staying close to the direction of steepest descent.

The procedure makes explicit use of the proportions of demands assigned on each path. This information represents a huge amount of data and, generally, is very difficult to obtain. Yet, the procedure may be easily performed by using the capabilities of STAN, a commercially-available software dedicated to the analysis of multicommodity, multimodal transportation systems. In STAN, the path demand proportions are considered implicitly and their explicit computation is not required. This implementation is presented next.

4 Implementation Using STAN

The procedure introduced in the previous section is efficiently implemented using the path analysis capabilities of the STAN software.

STAN (Strategic Transportation ANalysis) is an interactive-graphic transportation planning package designed for national or regional strategic analysis and planning of multimode multiproduct freight transportation (Guélat, Florian, and Crainic 1990; Crainic, Florian, and Léal 1990; Crainic *et al.* 1990; Crainic, Florian, and Larin 1994). The focus of the system is on strategic planning issues at the international, national , and regional levels, where the movements of several commodities through the transportation facilities and services of several carriers are considered simultaneously. The main questions addressed relate to the development of a given transportation system and its response to various modifications in its environment such as: infrastructure modifications, technology changes, evolution of demand, new government or industry policies, etc.

The STAN modelling framework is that of a multimodal network, made up of modes, nodes, links, and intermodal transfers, on which multiple products are to be moved by specific vehicles and convoys between given origin and destination points. It allows for the specification and combination of a wide variety of performance measures and assignment criteria, and makes use of the formulation in Section 2 to perform the assignment of demand to the network. The model and assignment algorithm are embedded in the STAN interactive-graphic system where they are complemented by a large number of tools to input, display, analyze, modify, and output data, and to specify the network and assignment models. Of these, particularly important for the adjustment of demand matrices are the *Matrix* and *Network Calculators*, and the *Path Analysis* capability. See Larin *et al.* (2000) for a detailed description of the STAN system, components, interfaces, and tools.

The Network Calculator allows the evaluation of algebraic expressions using as operands any permanent or temporary network attribute, including assignment results. It also allows the input of external data. In our case, it is used, for example, to input observed flow values and to compute predicted versus observed flow values. Algebraic expressions using matrices as operands may be evaluated cell-by-cell using the Matrix Calculator. Thus, the gradient and descent direction are computed using the Matrix Calculator. The Path Analysis module of STAN allows the efficient storage, retrieval, display, and recombination of information related to the paths generated during assignment. The procedure computes **implicitly** the proportions of demand assigned to each path using the step sizes saved at each iteration of the assignment. This information may then be used for a large gamut of analyses. Thus, in particular, path volumes may be saved in link and transfer attributes, partial demand matrices may be computed, and new path-related measures may be derived and aggregated into origin-destination matrices (Crainic *et al.* 1999, 2000). All these functionalities are required for an efficient implementation of the

procedure introduced in the previous section.

Using the assignment and analysis tools of STAN, each main iteration of the procedure corresponds to one assignment, two path analyses, plus a number of matrix and network calculations:

Initialization Solve the assignment with the initial demand matrix g^{c0}, saving the path information.

1. Mark
   (a) the links in $\widehat{\mathcal{A}}$ with observed volume $\hat{v}_a^c$;
   (b) the trasfers in $\widehat{\mathcal{T}}$ with $\hat{v}_t^c$;

   These two operations may be performed by using the Network Calculator to input the information in appropriately defined *tagged* link and transfer extra attributes.
2. $k = 0$.

Main Iteration Given g^{ck}, v_a^k, v_t^k, and w^{ck} from the previous iteration,

1. For each link (transfer) marked in Step 1, compute the $v_a^{ck} - \hat{v}_a^c$ ($v_t^{ck} - \hat{v}_t^c$) difference between the calculated and the observed flows and save the result in a link (transfer) *difference* extra attribute; This may be performed by using the Network Calculator;
2. Perform a path analysis for matrix g^{ck} with the difference link and transfer extra attributes both as path selection criteria and *additional path attribute*; Save the values of the additional path attributes in a $\breve{g}$ (weighted) *active demand* matrix:

$$\breve{g}_{od}^{ck} = \sum_{l \in \mathcal{K}_{od}^c} w_l^{ck} \left[\sum_{a \in \widehat{\mathcal{A}}} \delta_{al} (v_a^{ck} - \hat{v}_a^c) + \sum_{t \in \widehat{\mathcal{T}}} \delta_{at} (v_t^{ck} - \hat{v}_t^c) \right] \tag{23}$$

3. Use the Matrix Calculator to
   (a) Compute the gradient $b_{od}^{ck} = g_{od}^{ck} - \hat{g}_{od}^c + \breve{g}_{od}^{ck}$;
   (b) Compute the descent direction d^k;
   (c) Verify stopping criterion.
4. Perform a path analysis (same path selection criteria) for matrix d^k and save the corresponding $y_a^{ck}(d^k)$ and $y_t^{ck}(d^k)$ link and transfer volumes, respectively.
5. Compute λ^k by using a sequence of Network and Matrix calculations;
6. Use the Matrix Calculator to compute $g^{c(k+1)} = g^{ck} + \lambda^k d^{ck}$; $k = k + 1$.
7. Solve the assignment problem with demand matrix g^{ck} saving the path information, and go to Step 1.

This procedure assumes that the entire g^c matrix has to be adjusted, that is that the observed link and transfer flows may be a combination of volumes of all O-D pairs. When this is not true, the set $\mathcal{OD}^c$ is strictly smaller than the set of all cells with positive demand in g^c (as in the development of Section 2). A supplementary path analysis has to be performed then in order to obtain the g_{od}^{ck}, v_a^{ck}, and v_t^{ck} values that correspond to the $od \in \mathcal{OD}^c$.

The model and procedure described in the previous sections has been tested using data from an actual application of STAN. The network model represents the transportation system of a Brazilian state located in the North-East of the country. Eight (8) transportation modes are included: paved and unpaved roads, railway lines of 1.00m and 1.60m gauges, respectively, river and coastal navigation, port access, and connectors linking the zone centroids to the rest of the network. The network representation is made of 76 zones, 183 regular nodes, 866 modal links, and 1684 intermodal transfers. The demand of eight (8) product groups is to be assigned on this network according to a generalized cost formula that combines costs related to the operation of the system and the value of time required to perform the operations (including congestion-related delays).

The general methodology used to test the demand matrix adjustment method is the following:

1. Perform a multimode, multiproduct assignment with the current demand matrices.
2. Select a product and a subset of links and transfers with positive flow of that product. These will correspond to the *observed* flow values.
3. Perturb the demand matrix corresponding to the selected product. Perform an assignment using the perturbed matrix for the selected product (the rest of the assignment definition does not change). This yields the *predicted* flow volumes on the links and transfers of the network.
4. Execute the demand matrix adjustment procedure described above to adjust the predicted to the observed flows (and the perturbed matrix to the original one).

Ten (10) iterations of the STAN multimode, multiproduct assignment procedure have been used for the experiments described in this paper. This corresponds to an optimality gap of 0.3%. This indicates a solution which, without being optimal, is sufficiently close to this condition not to preclude a correct adjustment of demand matrices. We used product #1 for our experiments and selected 31 rail network links (the rail network includes the 1.00 and 1.60 meter gauge modes), and 45 transfers (each including at least one rail link in their definition). The matrix demand matrix for product #1 has 148 non-zero entries, or a density of 2.5%, which is a rather common situation in freight transportation planning.

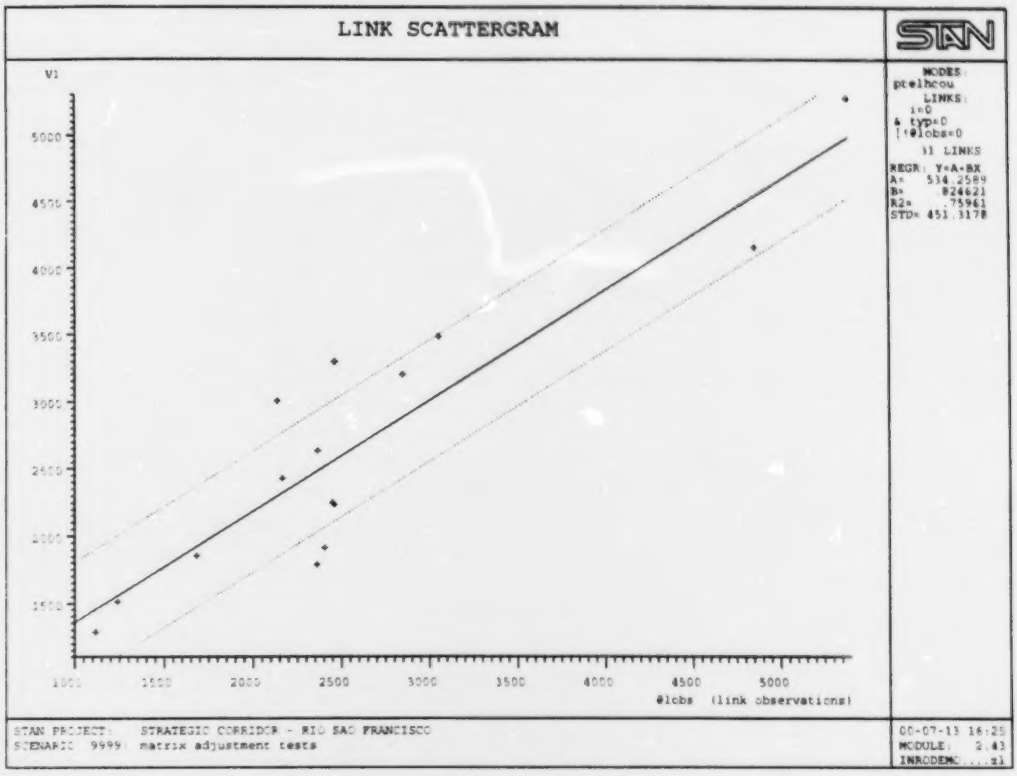

Figure 1: Initial Observed vs. Predicted Link Flows

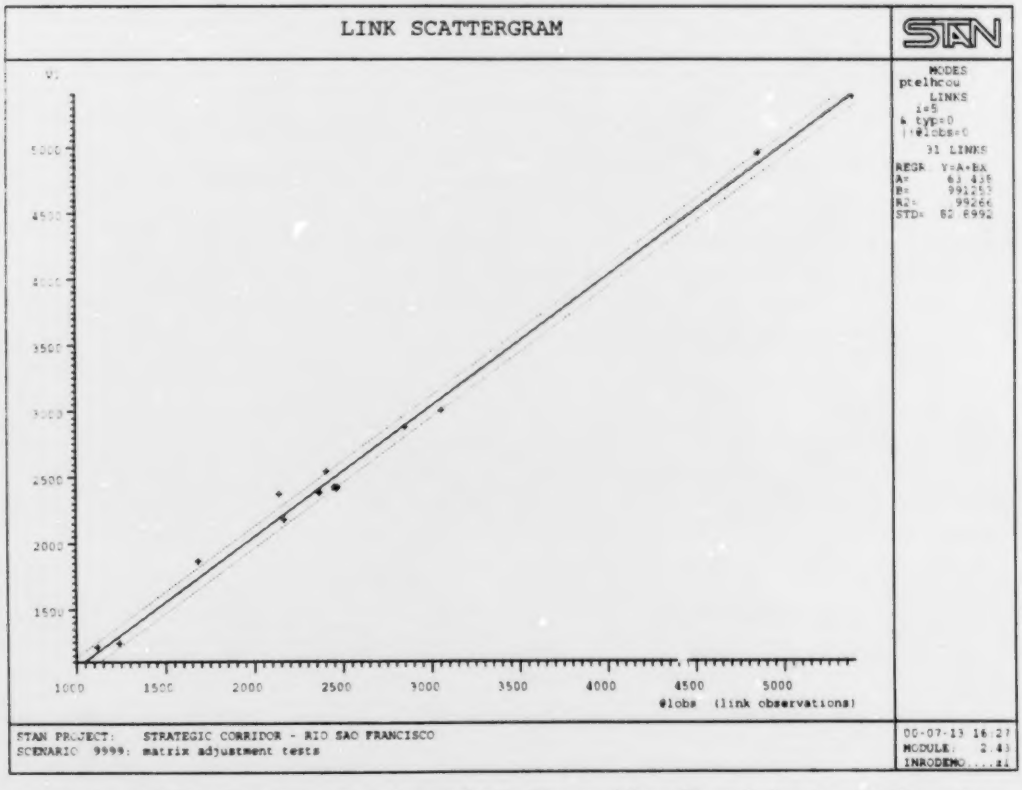

Figure 2: Observed vs. Predicted Link Flows After Five Adjustment Iterations

Several series of tests have been performed by varying the number of origin-destination pairs for which the demand is perturbed, as well as the amplitude of the perturbation. In the following, we present results for one of the extreme perturbation scenarios. Out of the 148 origin-destination pairs with positive demand, we perturbed 26, by a factor varying from -66.30% to 407.60% (for an average of 41.12%). This implied differences in observed versus predicted flows in the intervals $[-24.30\%,\ 40.55\%]$ and $[-58.76\%,\ 49,35\%]$ for links and transfers, respectively. Figure 1 illustrates the relationships between observed and predicted link flows following the perturbation of the demand matrix. A linear regression relation may be established, but the mean square regression coefficient, R^2, is not very good: $R^2 = 0.76\%$. The same relation for transfer flows yields $R^2 = 0.89\%$.

The demand matrix adjustment procedure has been implemented by using the macro language of the STAN software (Larin *et al.* 2000). This feature permits the implementation of complex iterative procedures by combining several data computation and analysis tools offered by the software.

Iteration	1	2	3	4	5
$G(g, w)$	1.3436	0.3507	0.1117	0.1131	0.0822

Table 1: Convergence of the Objective Function Values

The macro was run for five iterations. Table 1 illustrates the evolution of the objective function of the demand matrix adjustment model (9). One notices that the major adjustment is obtained, in this case, after the first iteration. The improvement continues, however, but by the third iteration, a satisfactory solution is essentially attained. In fact, one may notice that following the fourth iteration, the improvement is so small that numerical errors interfere with the convergence of the method. Figure 2 illustrates the observed versus predicted relation for link flows. The mean square regression coefficients are around 0.99% for both link and transfer flows.

5 Generalizations and Perspectives

Several generalizations of the model and procedures introduced in the previous sections may be of particular interest. In the following, we briefly examine the situation where observed flows exist for each of the products considered. This situation may be encountered when inquires at, for example, inspection stations or transshipment facilities, yield information specific for each product. The rapid penetration of Intelligent Transportation Systems technologies, especially transponders facilitating electronic pre-cleareance activities, will further contribute to the availibility of such detailed information.

Assume that for a subset of products $\hat{\mathcal{P}} \subseteq \mathcal{P}$, there are observed flows on some of the links and transfers of the network: $\hat{v}_a^p$, $a \in \hat{\mathcal{A}} \subseteq \mathcal{A}$ and $\hat{v}_t^p$, $t \in \hat{\mathcal{T}} \subseteq \mathcal{T}$, respectively. Let g^p be the current demand matrix for product $p \in \hat{\mathcal{P}}$ that is to be adjusted for a subset of origin to destination pairs $\mathcal{OD}^p$; denote $\mathcal{G} = \{g^p,\ m(p) \in \mathcal{M}^p,\ p \in \hat{\mathcal{P}}\}$. The goal is to adjust the $g^p \in \mathcal{G}$ matrices such that the volume of each corresponding product is as close as possible to the observed flows and the estimated matrices are as close as possible to the initial ones.

The formulation of the matrix adjustment problem (6) to (8) becomes

$$\text{Minimize } G(g, v) = \frac{1}{2} \sum_{p \in \hat{\mathcal{P}}} \left[\sum_{od \in \mathcal{OD}^p} (g_{od}^p - \hat{g}_{od}^p)^2 + \sum_{a \in \hat{\mathcal{A}}} (v_a^p - \hat{v}_a^p)^2 + \sum_{t \in \hat{\mathcal{T}}} (v_t^p - \hat{v}_t^p)^2 \right] \tag{24}$$

$$\text{s.t. } g_{od}^p \geq 0, \qquad p \in \hat{\mathcal{P}},\ od \in \mathcal{OD}^p \tag{25}$$

$$v = \arg\min \{F(g) \text{ constrained by (2) to (5)}\} \tag{26}$$

where relation (26) is equivalent to $v \equiv v^*$.

Similarly to (9), we may write equation (24) in terms of the path weights w_l^p, $l \in \mathcal{K}_{od}^p$, $od \in \mathcal{OD}^p$, $g^p \in \mathcal{G}$, as follows:

$$\text{Minimize } G(g, w) = \frac{1}{2} \sum_{p \in \hat{\mathcal{P}}} \left[\sum_{od \in \mathcal{OD}^p} (g_{od}^p - \hat{g}_{od}^p)^2 + \sum_{a \in \hat{\mathcal{A}}} \left(\sum_{od \in \mathcal{OD}^p} \sum_{l \in \mathcal{K}_{od}^p} \delta_{al} w_l^p g_{od}^p - \hat{v}_a^p \right)^2 + \sum_{t \in \hat{\mathcal{T}}} \left(\sum_{od \in \mathcal{OD}} \sum_{l \in \mathcal{K}_{od}^p} \delta_{tl} w_l^p g_{od}^p - \hat{v}_t^p \right)^2 \right] \tag{27}$$

The steepest descent algorithm of Section 3 then becomes:

Initialization Solve the lower level assignment problem (1) to (5) with initial demand matrices g^{p0} (usually, $g^{p0} \equiv g^p$, $g^p \in \mathcal{G}$). This yields initial link and transfer flows v^{p0} as well as corresponding path proportions w^{p0}.

Main Iteration Given g^{pk}, v_a^{pk}, v_t^{pk} and w_l^{pk} computed during the previous iteration:

1. Compute the gradient of $G(g, w_l^{pk})$ at g^k:

$$b_{od}^{pk} = \left.\frac{\partial G(g, w_l^{pk})}{\partial g_{od}^p}\right|_{g_{od}^{pk}} = g_{od}^{pk} - \hat{g}_{od}^p + \sum_{l \in \mathcal{K}_{od}^p} w_l^{pk} \left[\sum_{a \in \widehat{\mathcal{A}}} \delta_{al}(v_a^{pk} - \hat{v}_a^p) + \sum_{t \in \widehat{\mathcal{T}}} \delta_{tl}(v_t^{pk} - \hat{v}_t^p) \right], \quad od \in \mathcal{OD}^p,\ p \in \widehat{\mathcal{P}} \quad (28)$$

2. Compute the descent *gradient direction* d^k: $d_{od}^{pk} = -b_{od}^{pk} g_{od}^{pk}$.
   If $\nabla_g G(g^k, w^k) \cdot d^k \geq 0$, stop the algorithm. Otherwise
3. Compute the step length λ^k determined by

$$\lambda^k = \arg\min\{G(g^k + \lambda d^k) \text{ s.t. } \lambda b_{od}^{pk} \leq 1,\ od \in \mathcal{OD}^p,\ p \in \widehat{\mathcal{P}}\} \quad (29)$$

   which yields $\lambda^k = \alpha^k / \beta^k$, where

$$\alpha^k = \sum_{p \in \widehat{\mathcal{P}}} \sum_{od \in \mathcal{OD}^p} \left(\hat{g}_{od}^p - g_{od}^{pk}\right) d_{od}^{pk} + \sum_{p \in \widehat{\mathcal{P}}} \sum_{a \in \widehat{\mathcal{A}}} \left(\hat{v}_a^p - v_a^{pk}\right) y_a^{pk}(d^k) + \sum_{p \in \widehat{\mathcal{P}}} \sum_{t \in \widehat{\mathcal{T}}} \left(\hat{v}_t^p - v_t^{pk}\right) y_t^{pk}(d^k)$$

   and

$$\beta^k = \sum_{p \in \widehat{\mathcal{P}}} \sum_{od \in \mathcal{OD}^p} \left(d_{od}^{pk}\right)^2 + \sum_{p \in \widehat{\mathcal{P}}} \sum_{a \in \widehat{\mathcal{A}}} \left(y_a^{pk}(d^k)\right)^2 + \sum_{p \in \widehat{\mathcal{P}}} \sum_{t \in \widehat{\mathcal{T}}} \left(y_t^{pk}(d^k)\right)^2 \quad (30)$$

4. $g_{od}^{p(k+1)} = g_{od}^{pk} + \lambda^k d_{od}^{pk}$, $od \in \mathcal{OD}^p$, $p \in \widehat{\mathcal{P}}$; $k = k + 1$.
5. Solve the lower level assignment problem with demand matrix g^{pk}.
   Go to Step 1.

By using similar models and algorithms, one can tackle a large variety of other situations. A case in point occurs when, for example, only an aggregated observed measure (e.g., number of vehicles) is available for several products (see Annex). Other generalizations concern cases when the planner has varying degrees of confidence in the various data elements available. Then, weights can be associated to the problem elements, origin-destination pairs, products, matrices, to reflect these confidence levels and the limits on the amplitude of possible modifications to their values during adjustment.

Similar to the implementation described in Section 4, these procedures are reasonably easy to implement using the path analysis capabilities of STAN, even though the

multicommodity characteristic of the problems makes for more complex procedures and more demanding computations. Detailed descriptions and analyses of these procedures are, however, well beyond the scope of this paper.

It is worth noting that the freight demand matrix adjustment problem shares a number of complicating properties with the general multimodal, multiproduct assignment formulation. In particular, solutions are not unique. This means that the problem must be carefully set up (number and location of counting posts, type and nature of information gathered, coherence of countings, etc.) and results must be meticulously analyzed. In particular, one must be sure that the differences noticed between observed and predicted flows are not generated by other discrepancies such as errors in the zone or network definitions.

6 Conclusions

We have presented several models and algorithms for the adjustment of freight demand matrices to observed counts in the context of multimode, multiproduct transportation planning. We have also shown how this methodology can be easily implemented using a commercial planning software, STAN. The experimental results presented in the paper show that the methodology proposed is feasible, despite it apparent theoretical complexity.

We are now pursuing this line of work to explore more complex situations and to better characterize the best conditions of utilization of the methodology proposed.

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Annex - Aggregate Counts

Assume now that on some of the links and transfers of the network, one observes link and transfer flows $\hat{v}_a^c$, $a \in \widehat{\mathcal{A}} \subseteq \mathcal{A}$ and $\hat{v}_t^c$, $t \in \widehat{\mathcal{T}} \subseteq \mathcal{T}$, respectively, that correspond to an aggregation of the actual flows of a group of products $\widehat{\mathcal{P}} \subseteq \mathcal{P}$. Without loss of generality, assume summation as the aggregation operator. This corresponds, for example, to the very frequent case when vehicles are counted in a number of points of the network without specific information about the product transported.

Similarly to the preceding case, g^p stand for the demand matrices for product $p \in \widehat{\mathcal{P}}$ that are to be adjusted, but the procedure aims to modify these matrices such that the sum of the volumes of the corresponding products is as close as possible to the aggregated observed flows.

The matrix adjustment model (24) to (26) then becomes

$$\text{Minimize } G(g,v) = \frac{1}{2}\left[\sum_{p\in\widehat{\mathcal{P}}}\sum_{od\in\mathcal{OD}^p}(g_{od}^p - \hat{g}_{od}^p)^2 + \sum_{a\in\widehat{\mathcal{A}}}\left(\sum_{p\in\widehat{\mathcal{P}}} v_a^p - \hat{v}_a^c\right)^2 + \sum_{t\in\widehat{\mathcal{T}}}\left(\sum_{p\in\widehat{\mathcal{P}}} v_t^p - \hat{v}_t^c\right)^2\right] \quad (31)$$

subject to (25) and (26).

In terms of the path proportions w_l^p, $l \in \mathcal{K}_{od}^p$, $od \in \mathcal{OD}^p$, $p \in \widehat{\mathcal{P}}$, (31) becomes:

$$\text{Minimize } G(g,w) = \frac{1}{2}\left[\sum_{p\in\widehat{\mathcal{P}}}\sum_{od\in\mathcal{OD}^p}(g_{od}^p - \hat{g}_{od}^p)^2 + \sum_{a\in\widehat{\mathcal{A}}}\left(\sum_{p\in\widehat{\mathcal{P}}}\sum_{od\in\mathcal{OD}^p}\sum_{l\in\mathcal{K}_{od}^{m(p)}}\delta_{al}w_l^p g_{od}^p - \hat{v}_a^c\right)^2 + \sum_{t\in\widehat{\mathcal{T}}}\left(\sum_{p\in\widehat{\mathcal{P}}}\sum_{od\in\mathcal{OD}}\sum_{l\in\mathcal{K}_{od}^{m(p)}}\delta_{tl}w_l^p g_{od}^p - \hat{v}_t^c\right)^2\right] \quad (32)$$

The steepest descent algorithm just described may then be used with

$$b_{od}^{pk} = \left.\frac{\partial G(g, w^k)}{\partial g_{od}}\right|_{g_{od}^{pk}} = g_{od}^{pk} - \hat{g}_{od}^{c}$$

$$+ \sum_{l \in \mathcal{K}_{od}^{p}} w_l^{pk} \left[\sum_{a \in \hat{\mathcal{A}}} \delta_{al} \left(\sum_{p \in \hat{\mathcal{P}}} v_a^{pk} - \hat{v}_a^{c} \right) + \sum_{t \in \hat{\mathcal{T}}} \delta_{tl} \left(\sum_{p \in \hat{\mathcal{P}}} v_t^{pk} - \hat{v}_t^{c} \right) \right],$$

$$od \in \mathcal{OD}^p, \; p \in \hat{\mathcal{P}} \quad (33)$$

and $\lambda^k = \alpha^k / \beta^k$, where

$$\alpha^k = \sum_{p \in \hat{\mathcal{P}}} \sum_{od \in \mathcal{OD}^p} \left(\hat{g}_{od}^{p} - g_{od}^{pk} \right) d_{od}^{pk}$$

$$+ \sum_{a \in \hat{\mathcal{A}}} \left(\hat{v}_a^{c} - \sum_{p \in \hat{\mathcal{P}}} v_a^{pk} \right) \sum_{p \in \hat{\mathcal{P}}} y_a^{pk}(d^k) + \sum_{t \in \hat{\mathcal{T}}} \left(\hat{v}_t^{c} - \sum_{p \in \hat{\mathcal{P}}} v_t^{pk} \right) \sum_{p \in \hat{\mathcal{P}}} y_t^{pk}(d^k)$$

and

$$\beta^k = \sum_{p \in \hat{\mathcal{P}}} \sum_{od \in \mathcal{OD}^p} \left(d_{od}^{pk} \right)^2 + \sum_{a \in \hat{\mathcal{A}}} \left(\sum_{p \in \hat{\mathcal{P}}} y_a^{pk}(d^k) \right)^2 + \sum_{t \in \hat{\mathcal{T}}} \left(\sum_{p \in \hat{\mathcal{P}}} y_t^{pk}(d^k) \right)^2 . \quad (34)$$